A Petri Net Approach Based Elementary Siphons Supervisor for Flexible Manufacturing Systems

Mowafak Hassan Abdul-Hussin
Central Technical Information's & Communications,
University of Technology, Baghdad. IRAQ, E-mail: mow_abdul2007@yahoo.com

Abstract— This paper presents an approach to constructing a class of an S^3PR net for modeling, simulation and control of processes occurring in the flexible manufacturing system (FMS) used based elementary siphons of a Petri net. Siphons are very important to the analysis and control of deadlocks of FMS that is significant objectives of siphons. Petri net models in the efficiency structure analysis, and utilization of the FMSs when different policy can be implemented lead to the deadlock prevention. We are representing an effective deadlock-free policy of a special class of Petri nets called S^3PR. Simulation of Petri net structural analysis and reachability graph analysis is used for analysis and control of Petri nets. Petri nets contain been successfully as one of the most powerful tools for modelling of FMS, where Using structural analysis, we show that liveness of such systems can be attributed to the absence of under marked siphons.

Index Terms— deadlock prevention, simulation, elementary siphon, FMSs, Petri net, PN-Toolbox V. 2.3, S^3PR

1. Introduction

Recently manufacturing systems have emerged as a most important application area. Petri nets are well suited to model the complex interactions between the components of manufacturing systems, with a particular focus on applications in manufacturing. Flexible manufacturing system (FMS) is proposed to produce a set of different types of products of varying batch size. It contains a set of computer-controlled machines and transportation systems. Various types of raw components enter it and are processed concurrently. Hence, part resources among various competing jobs have to be carefully controlled and coordinated. For this purpose, a powerful tool for modeling it is called Petri nets (PNs) can be used. Petri net is a modeling tool applicable to numerous of FMS. Thus, system has been modeled with a Petri net, its logical behavior can be studied with a variety of techniques, such as invariant analysis, reachability graph generation, and simulation of FMS.

Due to the high capital involvement in an FMS, optimization of its efficiency is of particular importance. The following three groups of problems can be investigated: 1) design problem; 2) process planning problems, and 3) operational problems. While the first group deals with the selection, layout and integration of the FMS components, the next two groups are concerned with the utilization of the system. However, the solution to the process planning problems has an impact on the formulation of the operational problems and the solution to the design problems has been also influenced the solution to the process planning problems. These inherent relationships are requiring models with an integrated approach in order to solve both design and utilization problems of FMSs. Recently the usefulness of Petri net models to efficiently analyze FMS problems has been recognized. The chief advantage of Petri nets is that they provide a simple, abstract and general setting a formal method for specifying and verifying concurrent, investigating distributed systems is presented [4], i. e. analyzing, simulating and evaluating the system behaviour, where aspects of concurrency, commutation, synchronization and cooperation among subsystems are to be taken into account. As for the ability to represent the dynamic behaviour of the modeled system in both steady and transient state, and thus can be applied in the solving of FMS planning problems as well as to study the system control issues providing that the results achieved can be directly implemented in the Petri nets approach based elementary siphons controller.

Abdul-Hussin (2014) [1, 2] proposed two techniques when analysis Petri nets models for deadlock prevention in FMSs, structure analysis to find the elementary siphons, and reachability graph analysis was used Petri net with MATLAB. **S**iphons and traps are analysis structures which allow for some implications on the net's can be well controlled by adding control places and related arcs for strict minimal siphons (SMS) of its Petri net model for each un-controlled siphon in the net in order to become deadlock-free stations in the system. The structured analysis techniques and behavior in Petri nets are investigating the relationship between the behavior and structures of the net. In particular structural, a computer simulation system, and the Petri Net Toolbox in MATLAB [10] environment, which is based on analytical models of concurrent processes of the reusable resources that are aimed mainly at providing for control of a system is representative of this work.

International Journal of Scientific & Engineering Research Volume 6, Issue 5, May-2015
ISSN 2229-5518

Abdul-Hussin (2015) [3, 4] presents a structural analysis of Petri nets, where siphon is a main utility used in the development of Petri net theory to control of flexible manufacturing systems (FMSs) modeling, control and simulation, which has been exploited successfully for the design of supervisors of some supervisory control problems of FMS. In addition, the effective deadlock prevention policy of a special class of Petri nets namely S^3PR and be shown the discriminating siphon can be solved deadlock prevention policy. In addition, Petri net models in the efficiency structure analysis and utilization of the FMSs when different policy can be implemented for the prevention of deadlock.

Ezpeleta et al. [5] is first defines a class of Petri nets called system of simple sequential processes with resources (S^3PR. It can model a class of FMS in which a set of different types of products can be manufactured concurrently and each step in one manufacturing process only needs one resource such as a machine or robot. While the competition among manufacturing processes of the limited resources, deadlocks can occur. One way policy, which is based on strict minimal siphons (SMSs), is proposed to prevent deadlocks.

The theory of elementary siphons is first proposed by Li and Zhou [7–9] to overcome the structural complexity problem in a liveness-enforcing supervisor defined from the existing methods. The theory of elementary siphons has been mature1y used in ordinary Petri nets such as S^3PR [1–9] [11]. Due to the complex usage of shared resources in generalized Petri nets, elementary siphons in [5] are not sufficient to consider the weight information and multiple usage of shared resources. They are dividing siphon into two kind's elementary and dependent siphons. They can control all SMS by controlling elementary SMS only, leading to a simple supervisor, and this is needed control place smaller than [5]. In addition, they are used a linear integer programming (LIP) test must be carried out to decide the liveness of the controlled system. For all the deadlock prevention policies afterwards monitor added to SMS to enforce liveness, which is resulting of a deadlock-free Petri Net. This new method requires a much smaller number of control places. Although this paper explores the ways to minimize the new additions of places, while achieving the same controlling purpose, the control policy is similar to [5]. In this paper, an elementary siphon concept is used to reduce the number of control places. Moreover, they are proposing a method to compute some SMS in a S^3PR based on resource circuits. In their approach, for each resource circuit in the net, they compute its related strict minimal siphon.

In this paper, a deadlock prevention policy is proposed in a class of Petri nets called S^3PR. In addition, the structural analysis and reachability graph analysis are used for simulation, analysis and control of Petri nets.

Organization. In section 2, briefly review preliminaries to Petri nets that are used in this paper. Section 3, a method of computing all, the concept of elementary siphons in S^3PR is developed. Section 4, introduces the practical application of Petri net represented an FMS example. Finally, section 5, concludes this paper.

2. PRELIMINAIRS [4, 9]

Definition 1. A Petri net is a 4-tuple $\Psi = (P, T, E, W)$, where P, and T are finite, non-empty, and disjoint sets. P is a set of places, and T is a set of transitions with $P \cup T \neq \emptyset$, and $P \cap T = \emptyset$. $E \subseteq (P \times T) \cup (T \times P)$ is called a flow relation or the set of directed arcs. The net has represented by arcs with arrows from places to transitions or from transitions to places. W: $E \rightarrow Z^+$ is a mapping that assigns a weight to any arc, where $Z^+ = \{0, 1, 2, \dots\}$. $\Psi = (P, T, E, W)$ is ordinary, denoted as $\Psi = (P, T, E, W)$, (When Weights W, of the arcs (W) = l, the net Ψ is called ordinary Petri net). The Weights W: $(P \times T) \cup (T \times P) \rightarrow Z^+$ is a mapping that assigns a weight to an arc: $W(x, y) > 0$ if $(x, y) \in E$, and $W(x, y) = 0$ otherwise, where $x, y \in P \cup T$ and Z^+ denote the set of non-negative integers.

Definition 2. In the absence of self-loops, an equivalent information is given by the incidence matrix. The computation of the incidence matrix C is subtracting C^- from C^+ that is:
$C = (C^+ - C^-)$, or $C_{ij} = post(t_j, p_i) - pre(p_i, t_j)$.

A Petri net $\Psi = (P, T, E, W)$ can be alternatively represented by its flow matrix or incidence $C = (C_{ij})$ which is defined by: $C^+(P \times T) = W(E(P \times T))$ and $C^-(P \times T) = W(E(P \times T))$. Also we can write $C(p_i, t_j) = post(p_i, t_j) - pre(p_i, t_j)$, is the change in matrix C, where incidence matrix [C] of net Ψ is a $|P| \times |T|$ integer matrix and $[\Psi](p, t) = W(t, p) - W(p, t)$.

Definition 3. The preset of a node $x \in P \cup T$ is defined as $\bullet x = \{y \in P \cup T \mid (y, x) \in E\}$. While the post-set of a node $x \in P \cup T$ is defined as $x\bullet = \{y \in P \cup T \mid (x, y) \in E\}$. This notation can be extended to a set of nodes as: given $X \subseteq P \cup T$, $\bullet X = \cup_{x \in X} \bullet x$ and $X\bullet = \cup_{x \in X} x\bullet$. A marking is a mapping $M : P \rightarrow Z^+$, where $Z^+ \cup \{0\}$. M(p) denotes the number of tokens in place p. The preset (postset) of a set is defined as the union of the presets (postsets) of its elements. The pre- and post-sets of a transition $t \in T$ are defined respectively as:
$\bullet t = \{p \mid Pre(p, t) > 0\}$, and $t\bullet = \{p \mid Post(p, t) > 0\}$. The pre- and post-sets of a place $p \in P$ are defined respectively as:
$\bullet p = \{t \mid Post(p, t) > 0\}$ and $p\bullet = \{t \mid Pre(p, t) > 0\}$.

Definition 4. The pair (Ψ, M_0) is called a marked Petri net or a net system. The set of markings reachable from M in Ψ is denoted as $R(\Psi, M)$. A net (Ψ, M) is bounded iff $\exists k \in \Psi$, $\forall M \in R(\Psi, M_0)$, $\forall p \in P$, $M(p) \leq k$ holds. A transition $t \in T$ is enabled under M, denoted by $M[t\rangle$, iff $\forall p \in \bullet t$, $M(p) \geq 1$. A transition $t \in T$ is live under M_0 iff $\forall M \in R(\Psi, M_0)$, $\exists M \in R(\Psi, M_0)$, $M'[t\rangle$ holds. (Ψ, M_0) is deadlock-free iff $\forall M \in R(\Psi, M_0)$, $\exists t \in T$, $M[t\rangle$ holds. (Ψ, M_0) is live iff $\forall t \in T$, t is live under M_0.

Definition 5. A transition $t \in T$ is enabled at marking M, denoted by $M[t>$, iff $\forall p \in \bullet t$, $M(p) > 0$. An enabled transition t at M can be fired, resulting next marking M′, denoted by $M[t>M'$, where $M'(p) = M(p) - 1$. $\forall p \in \bullet t \setminus t\bullet$; $M'(p) = M(p) + 1$, $\forall p \in t\bullet \setminus \bullet t$; and otherwise $M'(p) = M(p)$, for all $p \in P$. A sequence of transitions $\sigma = t_1, t_2, t_3, \dots t_k$, where $t_i \in T$, and $i = 1, 2, \dots, k$, is feasible from a marking M, if $M_i[t_i > M_{i+1}$, $i = 1, 2, \dots, k$, where $M_1 = M$ and M_i's are called reachable mark-

International Journal of Scientific & Engineering Research Volume 6, Issue 5, May-2015
ISSN 2229-5518

ings from M. Let (Ψ, M_0) denote the set of all reachable markings of Ψ from the initial marking M_0. (Ψ, M_0) is bounded iff $\exists k \in Z \setminus \{0\}$, $\forall M \in$ (Ψ, M_0): $p \in P : M(p) \leq k$ holds. We assume that in this paper all Petri nets are bounded and pure.

Definition 6. A sequence of transitions $\sigma = \{t_1, t_2, \ldots, t_n\}$ is a firing sequence of (Ψ, M_0) iff there exists a sequence of markings such that is: $M_0[t_1\rangle M_1[t_2\rangle M_2, \ldots, [t_n\rangle M_n$. Moreover, marking M_n is said to be reachable from M_0 by firing σ, and this is denoted by $M_0[\sigma\rangle M_n$. The firing sequence is a marking (M_1, M_2, M_3, … , M_{n+1}) such that: ($\forall$ i, $1 \leq i \leq n$), and ($M_i[t_i\rangle M_{i+1}$), We can also write its by [$M_1[\sigma\rangle M_{n+1}$]. The set of all markings reachable from M_0 is denoted by reachability set R(M_0). The function σ': $T \to Z^+$ is the firing count vector of the firable sequence σ, i.e. σ′[t] represents the number of occurrences of $t \in T$ in σ. If $M_0[\sigma\rangle M'$, then we can write in vector form $M' = M_0 + C.\ \sigma'$, which is referred to as the linear state equation of the net. A marking M_0 is said to be potentially reachable iff $\exists X \geq 0$ such that: $M' = M_0 + C.\ \sigma \geq 0$, where σ is a firing sequence, a vector which is i-th denotes the number of occurrences of in σ. For $M_0[\sigma\rangle M_n$, we have $M_n = M_0 + C.\sigma_{ti}$, which is called the state equation of net Ψ, where σ, called the firing count vector, is a vector which is i-th entry denotes the number of occurrences of t_i in σ.

Definition 7. A P-vector is a column vector I: $P \to Z$ indexed by P, where Z is the set of integers. I is a P-invariant (place invariant) if and only if $I \neq 0$ and $I^T \bullet [\Psi] = 0^T$ holds. P-invariant I is said to be a P-semiflow if every element of I is non-negative. $||I||^+ = \{p \in P \mid I(p) \neq 0\}$ is called the support of I. If I is a P-invariant of (Ψ, M_0) then $\forall M \in R(\Psi, M_0)$: $I^T \bullet M = I^T \bullet M_0$. In an ordinary net, siphon S is controlled by P-invariant I under M_0 if and only if ($I^T \bullet M_0 > 0$) and $\{p \in P \mid I(p) > 0\} \subseteq S\}$. Such a siphon is called invariant-controlled siphons.

Definition 8. A nonempty set of places $S \subseteq P$ is a siphon (trap) iff $^\bullet S \subseteq S^\bullet$ (resp. $Q^\bullet \subseteq {^\bullet Q}$, for trap) holds. A siphon is said to be minimal if there is no siphon contained in S it as a proper subset. A minimal siphon that does not contain the support of any P-invariant is called a strict minimal siphon (SMS). A siphon S is controlled if it can never be emptied. It is said to be invariant-controlled by P-invariant I iff $I^T \bullet M > 0$ and $||I||^+ \subseteq S$. M(p) indicates the number of tokens in p under M. p is marked by M if M(p)>0. A subset $S \subseteq P$ is marked by M if at least one of its places is marked by M. The sum of tokens in all places in S is denoted by M(S), where M(s) = $\sum_{p \in S} M(p)$. A siphon S is said to be empty at marking M if M(S) = 0. A siphon is under marked if no transition in $S^\bullet$ can fire.

Definition 9. A PN is live under M_0 iff $\forall$ $t \in T$, t is live under M_0. A transition $t \in T$ is live under M_0 iff $\forall M \in R(\Psi, M_0)$, $\exists M' \in R(\Psi, M_0)$, t is friable under M′. A transition $t \in T$ is dead under M_0 if $\nexists M \in R(\Psi, M_0)$, where t is friable. A marking $M \in R(\Psi, M_0)$ is a (total) deadlock iff $\forall$ $t \in T$, t is dead.

Definition 10. (Conservative). A Petri Net PN with the initial marking M_0 is strictly a conservative, if for all markings M, elements of the Reachability set R(M_0), the number of tokens remains constants, i.e. A marked Petri net Ψ = (Ψ, M_0), is said to be conservative iff :

$$\sum_{i=1}^{n} M(p_i) = \text{constant} \quad \forall M \in R(M_0).$$

If a marked Petri net is conservative, then the sum of all tokens will remain a constant in all reachable markings.

$$\sum_{p_i \in p} M(p_i) = \sum_{p_i \in P} M_0(p_i).$$

A PT-net Ψ = (P, T, E, W) is said to be conservative if and only if there exists a |P|-vector x > 0 such that x . C = 0, where C is the incidence matrix of Ψ, for a p-invariant x and any markings M_i, $M_j \in \Psi^n$, which are reachable from M_0 by the firing of transitions, it holds. $X^T . M = X^T . M_0$.

Example 1. An example of a Petri net is shown in Fig. 1. Places are circles and transitions by rectangles or bars. This is the convention, which has been adopted for ordinary Petri nets when the weight of arcs equal's to one. The initial marking is M_0 = {1, 1, 1, 0, 0}. To design a controller to avoid deadlock, it is necessary to find the strict minimal siphons as: $S_1 = \{p_2, p_3, p_4\}$, $S_2 = \{p_1, p_3, p_4\}$, and the strict minimal trap as $Q_1 = \{p_2, p_3, p_4, p_5\}$, and $Q_2 = \{p_1, p_3, p_4, p_5\}$. Siphons are very important in the analysis and control of deadlocks to the Petri net modes of simulation are implemented in the **PN Toolbox with MATLAB**, [10].

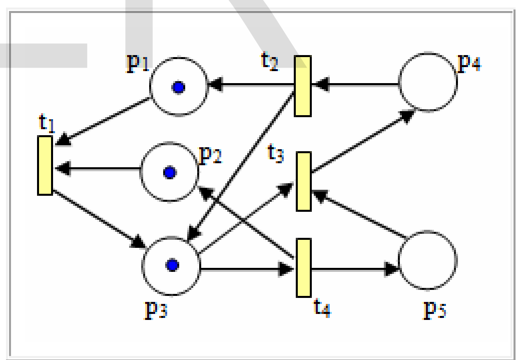

Fig. 1, shows a simple example of a Petri net.

Definition 11. The incidence matrix of Ψ is a matrix [Ψ]: $P \times T \to \{-1, 0, 1\}$ such that $[\Psi](p, t) = -1$, $p \in {^\bullet t} \setminus t^\bullet$; $[\Psi](p, t) = 1$, $p \in t^\bullet \setminus {^\bullet t}$; and otherwise $[\Psi](p, t) = 0$ for all $p \in P$ and $t \in T$. A nonzero |P|-vector I: $P \to Z^+$ is a P-invariant if $I \geq 0$ and $I^T * [\Psi] = 0^T$, where Z is the set of integers. The support of a P-invariant I is the set of places: $||I|| = \{p \in P \mid I(p) \neq 0\}$. A P-invariant I is minimal if there does not exist a P-invariant I′ such that $||I'|| \subset ||I||$.

Any Petri Net can be representation as an incidence matrix. The Petri net of Fig. 1, has incidence matrices shown in Fig. 2.

An important practical, applied definition 6, (linear state equation) is Petri net a transition t of an ordinary. Petri net is enabled if and only if each of its input places contains at least one token. In the example represented in Fig. 1, the initial marking $M_0 = [1, 0, 1, 0, 0]^T$ and the following transitions can

International Journal of Scientific & Engineering Research Volume 6, Issue 5, May-2015
ISSN 2229-5518

be fired starting from M_0.

		t_1	t_2	t_3	t_4
incidence	P1	-1	1	0	0
Matrix	P2	-1	1	0	0
C =	P3	1	0	-1	-1
	P4	0	-1	1	0
	P5	0	0	-1	1

Fig.2. The incidence matrix of Fig. 1.

Applied definition (6), the state equation is: $M' = M_0 + C.\sigma_i$, and taking the column vector of firing transition from the incidence matrix on Fig. 2. The marking can evolve $M_1 = [0, 1, 0, 1, 0]^T$ after the firing of t_4. Such that $M_1 = M_0 + C.\ \sigma_{t4} = M_0\ [1, 0, 1, 0, 0]^T + t_4\ [0, 1, -1, 0, 1]^T = [1,1,0,0,1] = M_1$. Each of these new markings represents a node of the reachability tree and used next-state of firing transition. In the sequel, the next marking can be found by adding last marking to the column of transition shown in Fig. 2, where transition t_1 is enabled. The new marking $M_2 = M_1 + C.\sigma_{t1} = [1,1,0,0,1] + [-1, -1, 1, 0, 0]^T = [0, 0, 1, 0, 1] = M_2$. The next firing transition is t_3, $M_3 = M_1 + C.\sigma_{t3} = [1,1,0,0,1] + [0, 0, 0, 1, -1]^T = [1,1,0,1,0] = M_3$. In addition, $M_4 = M_2 + C.\sigma_{t3} = [0, 0, 1, 0, 1] + [0,0,0,1,-1] = M_4 = [0,0,1,1,0]$. The rest of our mathematical computation of the net in fig. 1, can be analytically the reachability tree of a Petri net is shown in Fig. 3, used the PN Toolbox with the MATLAB philosophy [10].

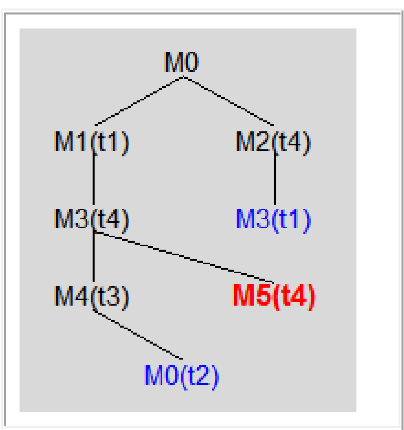

Fig. 3. Reachability tree of PN in MATLAB of Fig. 1

The Petri net model in Fig. 1 is non-live. We can see that the reachability tree is the deadlock shown in Fig. 3. The analysis Petri net in [10] MATLAB can be seen the deadlock is occurring at the marking M_5 in the red colored, when transition t_4 firing after this is t_4. The state of reachability tree is reached illustrated in Fig. 3 represents the deadlock shown in Fig. 1. In addition, a reachability tree is a fundamental tool to analyze various properties of a Petri net, such as liveness, deadlock, boundedness, (siphon), conservation, coverability.

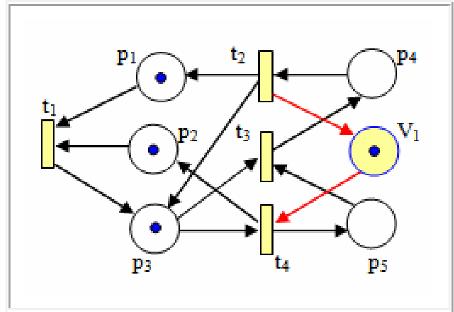

Fig. 4. Control Petri net of Fig. 1 by adding control places V_1 is in the red colored.

The final resulting controlled Petri net is shown in Fig. 5. It can be verified that deadlock does not occur in this Petri net. In the Fig. 1, the minimal siphon whose emptiness leads to the deadlock of the net shown in Figure 3. To control the net prevented from being unmarked, a place V_1 is added with $\bullet V_1 = \{t_2\}$ and $V_1\bullet = \{t_4\}$, as shown in Fig. 4, in order to the control is a Petri net in in this Petri net.

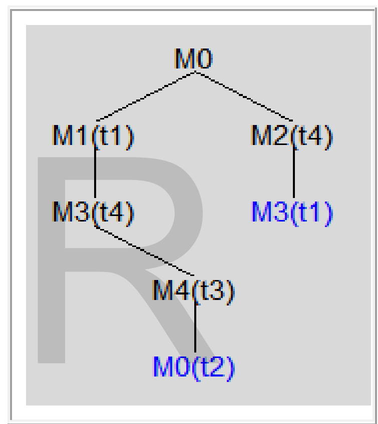

Figure 5: The coverability Fig 1.

The effecting controlled place in Petri net is shown in Fig. 5. It can be verified that deadlock does not occur tree is live.

Additional place V_1, called control place in order to controlled siphon. This example gives as a tool dynamic siphon is used for the purpose of control of the Petri net, to make a siphon controlled by adding a monitor.

3. Deadlock Prevent Policy

Deadlock prevention policy is dealing with a special class of Petri nets, which is a subclass of ordinary and conservative Petri nets called S^3PR. In this a section, we introduced some definitions have been needed for our application for a deadlock prevention policy that can keep the system in deadlock-free for a class of Petri nets, which is called S^3PR nets.

3.1. The Class of the S^3PR Net

The class of Petri nets investigated in this research is an S^3PR that is first proposed in Ezpeleta et al. [5]. Before the presentation of its formal definitions is needed for our application. The following results are mainly from [5, 6].

Definition 12. A simple sequential process (S^2P) is a Petri net

International Journal of Scientific & Engineering Research Volume 6, Issue 5, May-2015
ISSN 2229-5518

$\Psi = (P_A \cup \{p^0\}, T, E)$, where the following statements are true:
(1) $P_A \neq \emptyset$ is called a set of operation places;
(2) $p^0 \notin P_A$ is called the process idle place;
(3) A net Ψ is a strongly connected state machine; and
(4) Every circuit of Ψ contains place p^0.

Definition 13 . A system of simple sequential processes with resources (S^3PR): $\Psi = O_{i-1}^{k} \Psi_i = \Psi_i = (P_i \cup P_i^0 \cup P_R, T, E)$ is defined in [6] as the union of a set of nets: $\Psi_i = (P_i \cup \{P_i^0\} \cup P_R, T_i, E_i)$, sharing common places, where the following statements are true:

(1) P_i^0 is called the process idle places of net Ψ_i . The elements in P_A^i and P_R^i are called operation places and resource places respectively.

(2) $p_A^i \neq \emptyset$; $p_R^i \neq \emptyset$; $p_0^i \notin p_A^i$; and ($p_A^i \cup \{ p_0^i \} \cap p_R^i = \emptyset$; $\forall p \in p_A^i$, $\forall t \in {}^{\bullet}p$, $\forall t' \in p^{\bullet}$,
$\exists r_p \in p_R^i$, ${}^{\bullet}t \cap p_R^i = t'^{\bullet} \cap p_R^i = \{ r_p \}$;
$\forall r \in p_r^i$, ${}^{\bullet\bullet}r \cap p_A^i = r^{\bullet\bullet} \cap p_A^i \neq \emptyset$, and
${}^{\bullet}r \cap r^{\bullet} = \emptyset$, ${}^{\bullet\bullet}(p_i^0) \cap (p_R^i) = (p_i^0)^{\bullet\bullet} \cap p_R^i = \emptyset$;

(3) Ψ_i' is a strongly connected state machine, where $\Psi_i' = (p_A^i \cup \{p_0^i\}, T_i, E_i)$, is the resulting net after the places in p_R^i and related arcs are removed from Ψ_i .

(4) Every circuit of Ψ'_i contains place P_i^0;

(5) Any two Ψ'_i are composable when they share a set of common places. Every shared place must be a resource.

(6) Transitions in $(p_i^0)^{\bullet}$ *and* ${}^{\bullet}(p_i^0)$ are called source and sink transitions of the net Ψ respectively.

Definition 14. Let $\Psi_i = (P_A \cup P^0 \cup P_R, T, E)$, be an S^3PR. An initial marking M_0 is called an acceptable one if:
1) $\forall p \in P^0$, $M_0(p) \geq 1$; 2) $\forall p \in P_A$, $M_0(p) = 0$; and
3) $\forall p \in P_R$, $M_0(p) \geq 1$.

3.2. Elementary Siphon in Petri Nets

For the concepts elementary and dependent siphons are original work by Li et al. [6–9]. They are developed the Petri nets theory of computation and powerful mathematics. We have introduced the concept of elementary and dependent siphons as well as used in this paper.

Definition 15. Let $S \subseteq p$ is a subset of places of Petri net Ψ = (P, T, F, W). P-vector λ_S is called the characteristic P-Vector of S if and only if $\forall P \in S, \lambda_S(P) = 1$; otherwise $\lambda_S(P) = 0$.

Definition 16: η_S is characteristic T-Vector of S if and only if $\eta_S^T = \lambda_S^T[\Psi]$.

Definition 17. Let $S \subseteq P$ is a subset of places of Petri net Ψ. η_S is characteristic T-Vector of S if and only if $\eta_S = \lambda_S^T \bullet [C]$, where C is incidence matrix of the net Ψ.

Definition 18. Let Ψ = (P, T, F, W) is a net with $|P| = m$, $|T| = n$, and K siphons S_1– S_k, $k \in \Psi$. Let λ_{S_i} (η_{S_i}) is the characteristic P(T)-vector of siphon S_i, i $\in \Psi_k$. We define $[\lambda]_{k \times m}$ = $[\lambda_{S1} | \lambda_{S2} | \ldots | \lambda_{Sk}]^T$, and $[\eta]_{k \times n} = [\lambda]_{k \times m} \times [\Psi]_{m \times n} = [\eta_{S1} | \eta_{S2} | \ldots | \eta_{Sk}]^T$. Where $[\lambda]([\eta])$ is called the characteristic P(T)-vector matrix of the siphons in net Ψ.

Definition 19. Let $\eta_{S_\alpha}, \eta_{S_\beta}, \ldots, and\, \eta_{S_\gamma}$ =({ α, β, … , γ} $\subseteq \Psi_k$) a linearly independent maximal set of matrix $[\eta]$. Then $\Pi_E = \{S_\alpha, S_\beta, \ldots, S_\gamma\}$ is called a set of elementary siphons in net Ψ.

Definition 20. Let $S \notin \Pi_E$ be a siphon in net Ψ. Then S is called a strongly dependent siphon if η_S = $\sum_{S_i \in \Pi_E} a_i \eta_{S_i}$ holds, where $a_j \geq 0$.

Definition 21. The siphon $S \notin \Pi_E$ is called a weakly dependent siphon if ∃ non-empty $S^A, S^B \subset \Pi_E$, such that $S^A \cap S^B = \emptyset$, and $\eta_S = \sum_{S_i \in S^A} a_i.\eta_{S_i} - \sum_{S_i \in S^B} a_i.\eta_{S_i}$, where $a_i > 0$.

Let S be a (strongly or weakly) dependent siphon. To completeness, elementary siphons characteristic vectors:
if η_S can be linearly represented by elementary siphon's characteristic T-vectors $\eta_{S_1}, \eta_{S_2}, \ldots, \eta_{S_n}$ with non-zero coefficients, we say that S_1–S_n are the elementary siphons of S. Let Π be the set of siphons in which we are interested in a net. The sets elementary and dependent ones within the scope of Π, a set of siphons, are denoted by Π_E and Π_D, a respectively. Surely, the elementary siphons has $\Pi = \Pi_E \cup \Pi_D$.

Theorem 1. Let Ψ_{ES} be the number of elementary siphons in Ψ = (P, T, E, W). Then we have $\Psi_{ES} < \min\{|P|, |T|\}$. This result indicates that the smaller of a place and transition counts bound the number of elementary siphons in a net.

Theorem 2. Let S be a siphon if net Ψ = (P, T, E, W) and η_S be its characteristic T-vector. We can conclude that $\{t \in T | \eta_S(t) > 0\}$, $\{t \in T | \eta_S(t) = 0\}$, and $\{t \in T | \eta_S(t) < 0\}$ are a sets of transitions that is firings will increase, maintain, and decrease the number of tokens marked in S respectively.

International Journal of Scientific & Engineering Research Volume 6, Issue 5, May-2015
ISSN 2229-5518

4. Application of Petri Net for FMS

A flexible manufacturing cell as shown in Fig. 6 has two machine M1 and M2. Each machine tool can hold two parts at the same time. In addition, the cell contains two robots R1 and R2 and each of them can hold one part. Parts enter the cell through two loading buffers I1 and I2, and leave the cell through two unloading buffers O1 and O2. Two part types J1 − J2 are produced in parallel processes. The machine tools perform operations on raw parts and the robots deal with the movements of the parts.

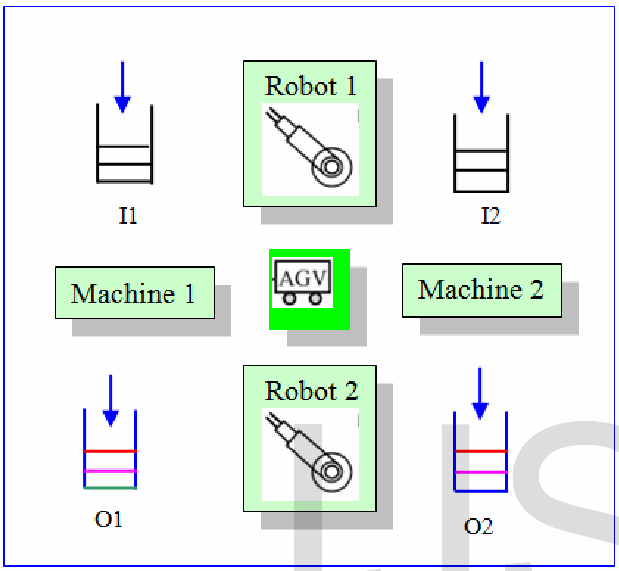

Figure 6. Layout of a manufacturing cell

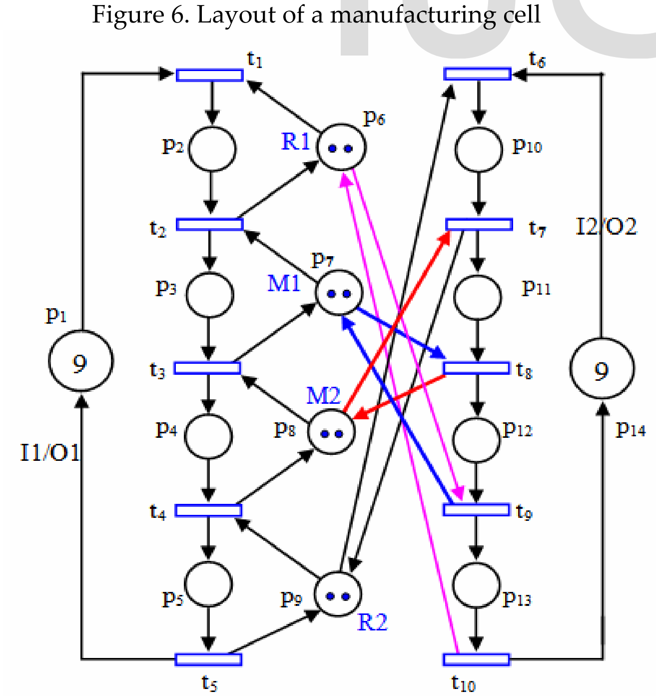

Fig. 7. An S^3PR (Ψ, M_0)

The Petri net model of the FMS is shown in Fig. 7, in which places p_6, p_9, denote R1, R2, and p_{10}, p_{11}, denote M1, and M2, respectively. Places p_1 and p_{14} are idle places, and $(p_2 – p_3)$ are operation places for production Line_1. The places $(p_{10} – p_{13})$ are presented operation places for production Line_2. The Petri net shown in Fig. 7 is represented an S^3PR by this place specification.

In order to transport the parts in the cell, a robot R1 can pick up and deliver products of the M1. Robot R1 holds the work-piece from store (p_1) and put in the buffer (p_2), then M1 process workpiece in place (p_3), and release to process two in M2, put in the buffer at (p_5). Either M1 or M2 may fail while processing a job. Robot R2 is unloading product and release. Similarly, for the production Line_2 for Part2. The Petri Net of Figure 7, models the production plans of each part type (P1 and P2) in the manufacturing cell of Fig. 6.

The net system is an S^3PR, and contains deadlocks. Simulation and structure analysis of the Petri net model used PN tool with MATLAB [10] and the reachability tree has the deadlock marking occurred at marking:

$M_{495} = [5,2,2,0,0,0,0,0,0,0,2,2,0,0,5]$,
$M_{581} = [4,2,2,1,0,0,0,0,0,0,2,1,0,0,6]$,
$M_{624} = [6,2,1,0,0,0,0,0,0,0,2,2,1,0,4]$,
$M_{674} = [3,2,2,2,0,0,0,0,0,0,2,0,0,0,7]$,
$M_{695} = [5,2,1,1,0,0,0,0,0,0,2,1,1,0,5]$,
$M_{748} = [7,2,0,0,0,0,0,0,0,0,2,2,2,0,3]$,
$M_{795} = [4,2,1,2,0,0,0,0,0,0,2,0,1,0,6]$, and
$M_{821} = [6,2,0,1,0,0,0,0,0,0,2,1,2,0,4]$.

The reachability graph has (1176) states with the initial marking. As a particular important application of our approach used an S^3PR to solve deadlock in FMS, where processes are executed concurrently and share a set of common resources. We consider Petri nets, which are obtained by asynchronous composition and are important to identify the minimal siphons. The Petri net in Fig. 7, has 12 strict minimal siphons:
$S_1 = \{p_3, p_6, p_7, p_{13}\}$, $S_2 = \{p_4, p_7, p_8, p_{12}\}$, $S_3 = \{p_4, p_6, p_7, p_8, p_{13}\}$, $S_4 = \{p_5, p_8, p_9, p_{11}\}$, $S_5 = \{p_5, p_7, p_8, p_9, p_{12}\}$,
$S_6 = \{p_5, p_6, p_7, p_8, p_9, p_{13}\}$, $S_7 = \{p_5, p_9, p_{10}\}$, $S_8 = \{p_{10}, p_{11}, p_{12}, p_{13}, p_{14}\}$, $S_9 = \{p_2, p_6, p_{13}\}$, $S_{10} = \{p_4, p_8, p_{11}\}$, $S_{11} = \{p_3, p_7, p_{12}\}$, $S_{12} = \{p_1, p_2, p_3, p_4, p_5\}$.

The set of siphons from $S_7–S_{12}$ is both a siphon and a trap. We have choice strict minimal siphons $S_1– S_6$ that do not contain any trap. The strict minimal siphons are be used:
$S_1 = \{p_3, p_6, p_7, p_{13}\}$, $S_2 = \{p_4, p_7, p_8, p_{12}\}$,
$S_3 = \{p_4, p_6, p_7, p_8, p_{13}\}$, $S_4 = \{p_5, p_8, p_9, p_{11}\}$,
$S_5 = \{p_5, p_7, p_8, p_9, p_{12}\}$, $S_6 = \{p_5, p_6, p_7, p_8, p_9, p_{13}\}$.

We are choice three the strict minimal such as:
$\lambda_{S1} = p_3 + p_6 + p_7 + p_{13}$,
$\lambda_{S2} = p_4 + p_7 + p_8 + p_{12}$, and
$\lambda_{S3} = p_4 + p_6 + p_7 + p_8 + p_{13}$.

The linearly independent vectors can be constructed in [η] shown as follows.
$\eta_{S1} = − t_1 + t_5 − t_7 + t_{10}$,
$\eta_{S1} = − t_1 + t_4 − t_6 + t_9$, and
$\eta_{S3} = − t_2 + t_3 − t_7 + t_8$.
In addition, the linearly independent T-vectors, we can be

International Journal of Scientific & Engineering Research Volume 6, Issue 5, May-2015
ISSN 2229-5518

constructed in [η] shown as follows:

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
η_{S1} =	-1	0	0	0	1	0	-1	0	0	1
η_{S2} =	-1	0	0	1	0	-1	0	0	1	0
η_{S3} =	0	-1	1	0	0	0	-1	1	0	0
η_{S4} =	-1	-1	1	1	0	-1	-1	1	1	0
η_{S5} =	-1	-1	1	0	1	0	-2	1	0	1

In order to find the control place VS_4, and VS_5, where $\eta_{S4} = \eta_{S2} + \eta_{S3}$, and $\eta_{S5} = \eta_{S1} + \eta_{S3}$: that is

$\eta_{S4} = -t_1 - t_2 + t_3 + t_4 - t_6 - t_7 + t_7 + t_8 + t_9$,

$\eta_{S5} = -t_1 - t_2 + t_3 + t_5 - 2t_7 + t_8 + t_{10}$.

The elementary siphon is an identification algorithm proposed in [8], $\Pi_E = (S_1, S_2, S_3)$ is a set of elementary siphons and S_4 and S_5 are dependent ones.

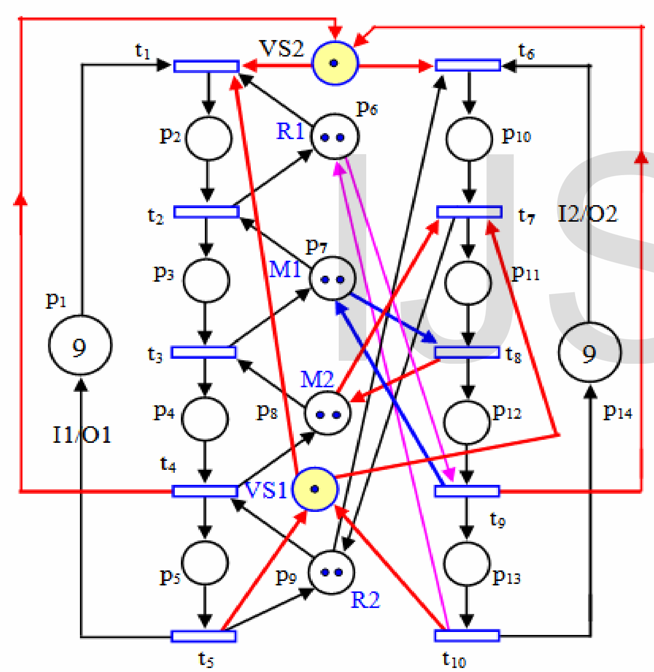

Fig. 8. Deadlock-free Petri net supervisor of S³PR net.

The additional controlled places net of Fig. 8, is live and obtain to the final marking M_{10}, and has 11 reachable states marking shown in Fig. 9, so that the Petri net is deadlock-free actually live. Siphons are generating even though the monitors are added. This example gives the experience utilization laboratory of Petri nets and used a simulation by computer Pentium CORE i5 Laptop, in order to demonstrate that Petri net have the ability to get a solution for the problems in FMS. In this experiment, we can add two-controlled net, which has only (11) reachable markings shown in Fig. 9, without needing third or more control places, because, the result is not affected to the net. In addition, only two control places are necessary, which actually coincide with the first two control places added by the previous method.

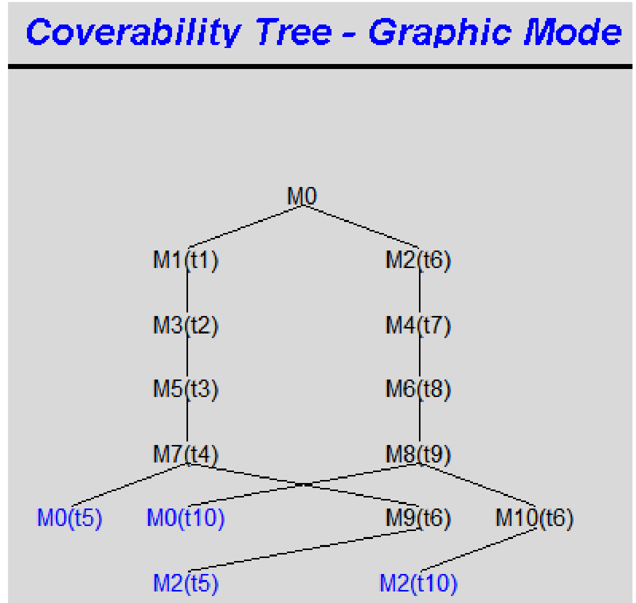

Fig. 9. Reachability tree of PN in MATLAB of Fig. 8.

From Fig. 8, we can see there are four resources exist in this system, leading to four minimal P-invariants, we would like to mention as follow:

$I_6 = p_2 + p_6 + p_{13}$, where, $M_0(p_6) = 2$,
$I_7 = p_3 + p_7 + p_{13}$, where $M_0(p_7) = 2$,
$I_8 = p_4 + p_8 + p_{11}$, where $M_0(p_8) = 2$,
$I_9 = p_5 + p_9 + p_{10}$, where $M_0(p_9) = 2$.

Let us come back to the Fig. 8, according to definition 7, there are many P-invariant can be found. For the strict minimal siphons, which can be emptied. There are $S_1 = \{p_8, p_{10}, p_{11}, p_{13}\}$, $S_2 = \{p_7, p_9, p_{10}, p_{14}\}$ and $S_3 = \{p_4, p_6, p_7, p_8, p_{13}\}$. For the purpose control place VS_1 is added such that: a control place VS_1 is added according to:

$I_1 = (0,1,1,1,1, 0, 0, 0, 0, 0,1,1,1,0,1VS_1)$, is a P–invariant of the resultant net (Ψ_1, M_1). Therefore, $I_1 \cdot M_1 = 0$ by definition 7, and it is easy to compute that: $[\Psi_1](VS_1, t) = -t_1 + t_5 - t_7 + t_{10}$. Similarly, $I_2 = (0,1,1,1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1VS_2)$, is also a P-invariant of the resultant net (Ψ_1, M_1). Hence $(I_2 \cdot M_1) = 0$, and $[\Psi_1] (VS_2, t) = -t_1 + t_4 - t_6 + t_9$. Similarly,
$I_3 = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1VS_3)$ is also a P-invariant of the resultant net (Ψ_1, M_1). Hence $(I_3 \cdot M_1) = 0$, and $[\Psi_1] (VS_3, t) = -t_2 + t_3 - t_7 + t_8$.

At the first, we can add the two control places VS_1, and VS_2 for the net (Ψ, M_0). We can see on the screen where Petri net is a simulation in [10] MATLAB running, that the net (Ψ_1, M_1) is deadlock-free (actually live), and the reachability graph has 11 states can be seen in Fig. 9.

For elementary siphons S_1 and S_2, the two monitor VS_1 and VS_2 are added respectively to the net of Fig. 8. We can apply definition 7, to find a P-invariant controlled of the Petri net depending of the incidence matrix existing in Fig. 8, implementation manuals.

International Journal of Scientific & Engineering Research Volume 6, Issue 5, May-2015
ISSN 2229-5518

For instance, Fig. 8 has shown an S^3PR net. $S_1 = \{p_3, p_6, p_7, p_{13}\}$ is a siphon of the net. We can apply **definition 7**, to find a P-invariant of the net in Fig. 8. This is:
$I_4 = (1, 1, 1, 0, -1, 1, 1, 0, -1, 0, 0, 1, 1, 1, -1VS_1)^T =$
$\{(p_3 + p_6 + p_7 + p_{13}) + p_1 + p_2 - p_5 - p_9 + p_{12} + p_{14} - 1VS_1\}$ is a P–invariant of (Ψ_1, M_1). Where, $I^T \bullet M_0 = [\{M(p_3) + M(p_6) + M(p_7) + M(p_{13})\} + M(p_1) + M(p_2) - M(p_5) - M(p_9) + M(p_{12}) + M(p_{14}) - M(VS_1)] = 1 > 0$. So that $||I_4||^+ = \{p_3, p_6, p_7, p_{13}\} \subseteq S$. Thus, $S_1 = \{p_3, p_6, p_7, p_{13}\}$is an invariant-controlled siphon and it can never be emptiable.

Similarly, for $S_2 = \{p_4, p_7, p_8, p_{12}\}$ is a siphon of the net. Also, $I_5 = (0, -1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, -1VS_2)^T = \{(p_4 + p_7 + p_8 + p_{12}) - p_2 - p_4 + p_{11} + p_{13} + p_{14} - 1VS_2\}$ is a P–invariant of (Ψ_1, M_1). Where, $I^T \bullet M_0 = [\{M(p_4) + M(p_7) + M(p_8) + M(p_{12})\} - M(p_2) - M(p_4) + M(p_{11}) + M(p_{13}) + M(p_{14}) - M(VS_2)] = 1 > 0$. So that $||I_5||^+ = \{p_4, p_7, p_8, p_{12}\} \subseteq S$. Thus, $S_2 = \{p_4, p_7, p_8, p_{12}\}$ is an invariant-controlled siphon and it can never be emptiable.

Similarly, for $S_3 = \{p_4, p_6, p_7, p_8, p_{13}\}$, is a siphon of the net.
$I_6 = (-1, 0, -1, 0, -1, 1, 1, 1, 0, 0, 0, 1, 1, 0, -1VS_3)^T = \{(p_4 + p_6 + p_7 + p_8 + p_{13}) - p_1 - p_3 - p_4 - p_5 + p_{12} - 1VS_3\}$ is a P–invariant of (Ψ_1, M_1). Where, $I^T \bullet M_0 = [\{M(p_4) + M(p_6) + M(p_7) + M(p_8) + M(p_{13})\} - M(p_1) - M(p_3) - M(p_4) - M(p_5) + M(p_{12}) - M(VS_2)] = 1 > 0$. So that $||I_6||^+ = \{p_4, p_6, p_7, p_8, p_{13}\} \subseteq S$. Thus, $S_3 = \{p_4, p_6, p_7, p_8, p_{13}\}$ is an invariant-controlled siphon and it can never be emptiable. This approach to P-invariant is used in order to add a monitor to every minimal siphon that can be emptied and the main objective is to add a certain area of the monitors will point to source transition.

5 Conclusion

The usefulness of the structure theory of Petri nets observed deadlock prevention for a class of FMS indicates that the number of additional monitors equals that of strict minimal siphons in an S^3PR net has been simulated. The deadlock prevention policy is presented in this paper, which is based on elementary siphon concepts. Therefore, the elementary siphons control phases, an ordinary control place is added to the original net for every elementary siphon to prevent the elementary siphons from being emptied, and a generalized control place is added to the modified net with its output arcs to the source transitions of the resultant net in generalized siphons control phase. A Petri net is a capability of hardware systems far exceed the sophistication of software systems needed for optimum control. This research is an attempt to advance software control capabilities of flexible manufacturing systems (FMSs). In addition, an FMS controller structure, has been designed and a software system conforming to this structure has been designed and implemented. A Petri net is efficient techniques to enumerate and reduce the reachability graph by adding monitors to all basic siphons and only those compound siphons.

References

[1] Abdul-Hussin Mowafak (2014)," Siphons and Traps Structural Analysis Techniques Behaviour of a Petri Nets", In: World Symp. on Computer Networks and Information Security, Intnational Conference on Advances in Enginerring & Technology (ICAET'2014), Hammamet, Tunisia, pp. 186-193.

[2] Abdul-Hussin Mowafak, (2014) ," Petri Nets approach to simulate and control of Flexible Manufacturing Systems", In: World Congress on E-learning, Education and Computer Sci., Intnational Conference on Advanced Studies in Computer Sci. and Eng., (ICASCSE'2014), Hammamet, Tunisia, pages 75-85.

[3] Abdul-Hussin Mowafak (2015), "Design of a Petri Net Based Deadlock Prevention Policy Supervisor for S^3PR", in Proc. IEEE-ISMS2015, 6th Intnational Conference on Intelligent Systems, Modelling and Simulation, Kuala Lumpur, Malaysia, pp. 46-52.

[4] Abdul-Hussin Mowafak (2015), " A Structural Analysis of Petri Nets-Based Siphons Supervisors of Flexible Manufacturing Systems", Published in: Proc. IEEE-UKSim-AMSS, 17th International Conference on Computer Modelling and Simulation, Cambridge University, March 2015, pp. 235-241.

[5] Ezpeleta. J; J.M. Colom, et al., (1995)," A Petri net based deadlock prevention policy for flexible manufacturing systems," IEEE Trans. on Robotics and Automation, vol. 11, n. 2, pp. 173-184

[6] Hou, YiFan, et al. (2014)," Extended Elementary Siphons and Their Application to Liveness-Enforcement of Generalized Petri Nets", Asian Journal of Control, Vol. 16, No. 5, pp. 1–22,

[7] Li ZhiWu, Zhou M. C., (2004), "Elementary siphons of Petri nets and their application to deadlock prevention in flexible manufacturing systems", IEEE Transactions on System Man,and Cybernetic. Part A: System and Humans, vol. 34, n. 1, pp. 38-51

[8] Li ZhiWu and M. C. Zhou, (2008),"Control of elementary and dependent siphons in Petri nets and their application," IEEE Trans. Syst., Man, Cybernetics Part A, System, Humans, vol. 38, no. 1, pp. 133–148

[9] Li ZhiWu and M. C. Zhou, "Deadlock Resolution in Automated Manufacturing Systems: A Novel Petri Net Approach. London, U.K. : Springer. 2009

[10] Mahulea, C., M. H. Matcovschi and O. Pastravanu, (2003)." Home Page of the Petri Net" PETRI NET Toolbox for MATLAB Version 2.3, http://www.ac.tuiasi.ro/pntool

[11] Wang ShouGuang, MengChu Zhou, et.al. (2015)," Design of a Maximally Permissive Liveness-Enforcing Supervisor with Reduced Complexity For Automated Manufacturing Systems", Asian Journal of Control, Vol. 17, No. 1, pp.190-201.

Ass. Prof. Dr. Mowafak H. Abdul-Hussin received the B.S. (1977) from the University of Al-Mustansiriyah, Baghdad and M.S., and Ph.D. degrees from the Technical University of Wroclaw, Poland, 1984, and 1989 respectively. Since 1991, he has been with University of Technology, Baghdad. His research interests include Petri net theory and applications of the Flexible Manufacturing Systems.